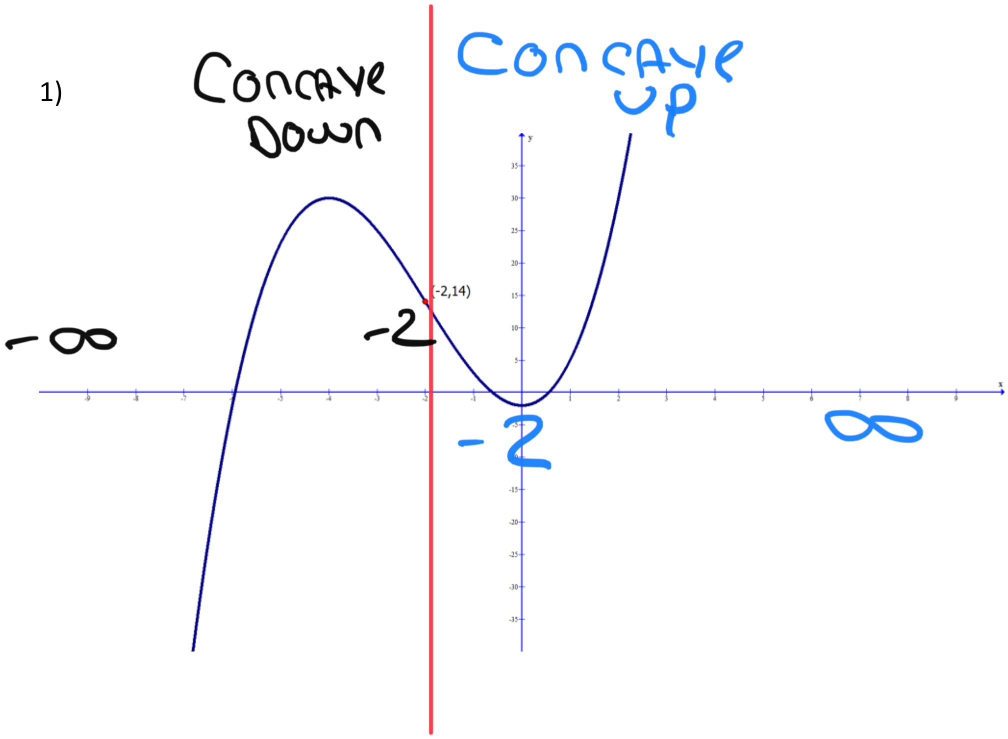
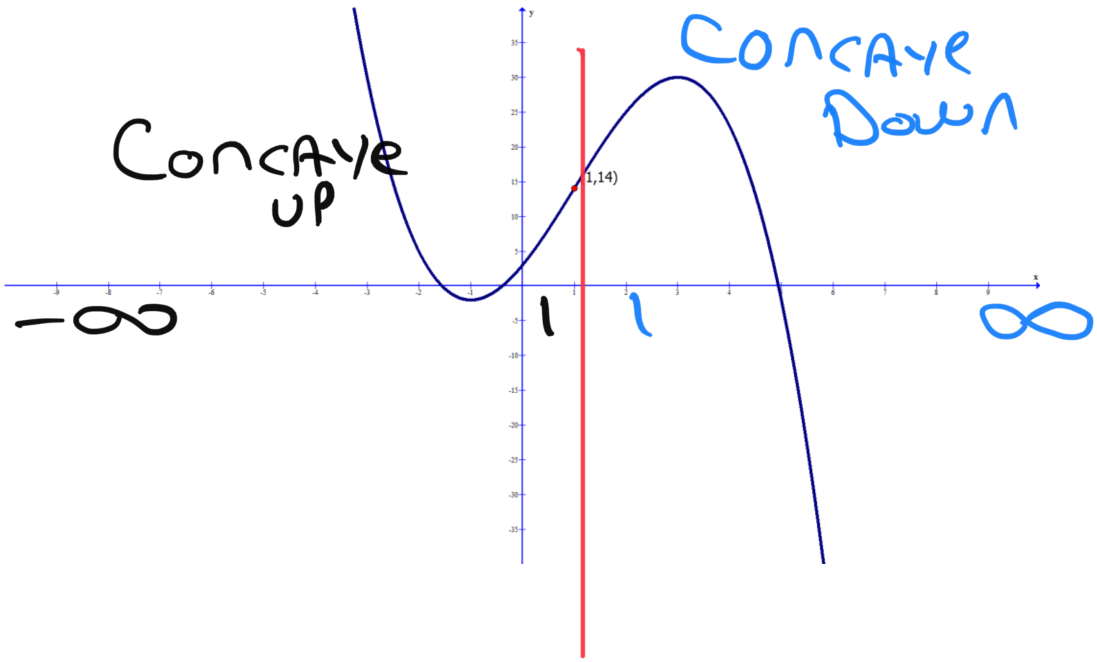
Section 3.3 Concavity and the Second Derivative Test
(Minimum Homework: 1 – 24 odds)

1)



1a) Find the open interval(s) where the graph of the function is concave up $(-2, \infty)$
1b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, -2)$
1c) Find all inflection points $(-2,14)$

3)

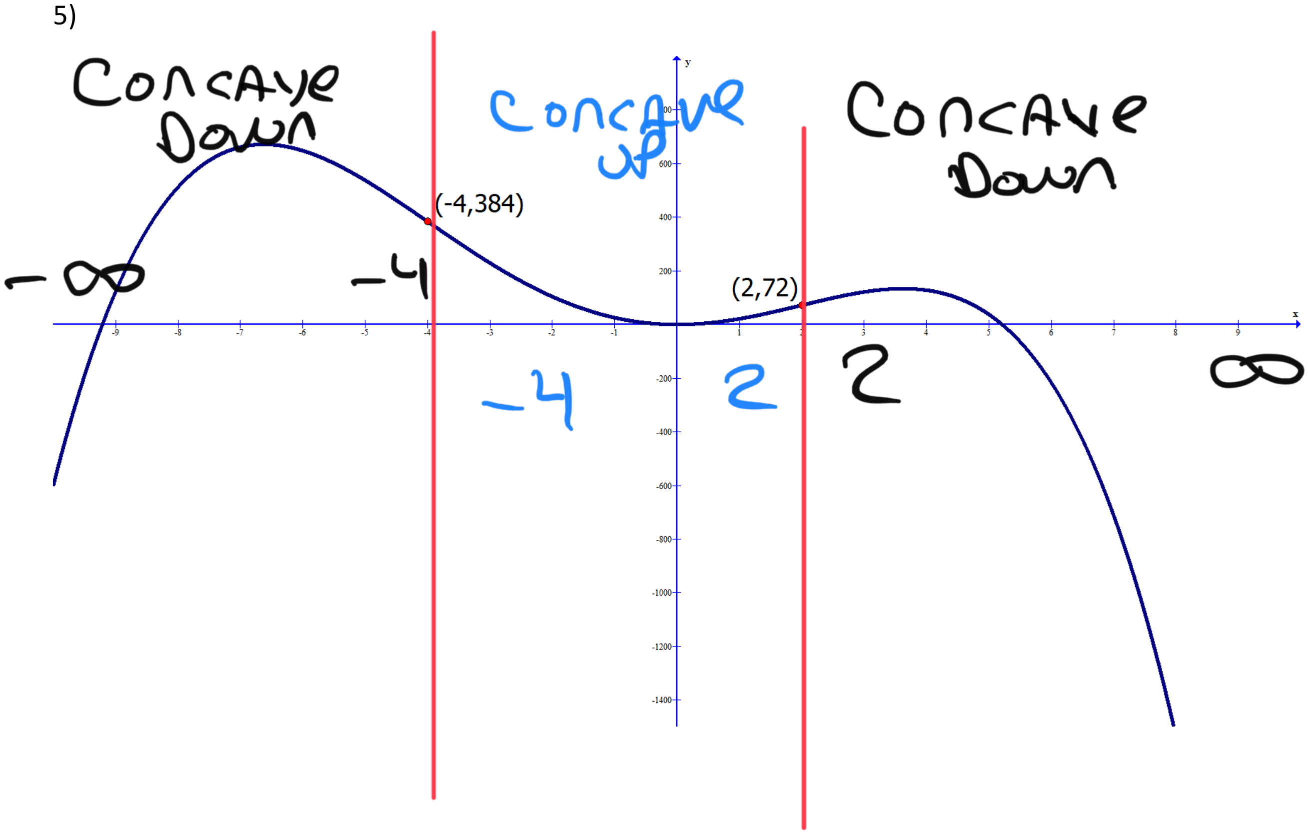


3a) Find the open interval(s) where the graph of the function is concave up $(-\infty, 1)$

3b) Find the open interval(s) where the graph of the function is concave down. $(1, \infty)$

3c) Find all inflection points $(1,14)$

5)

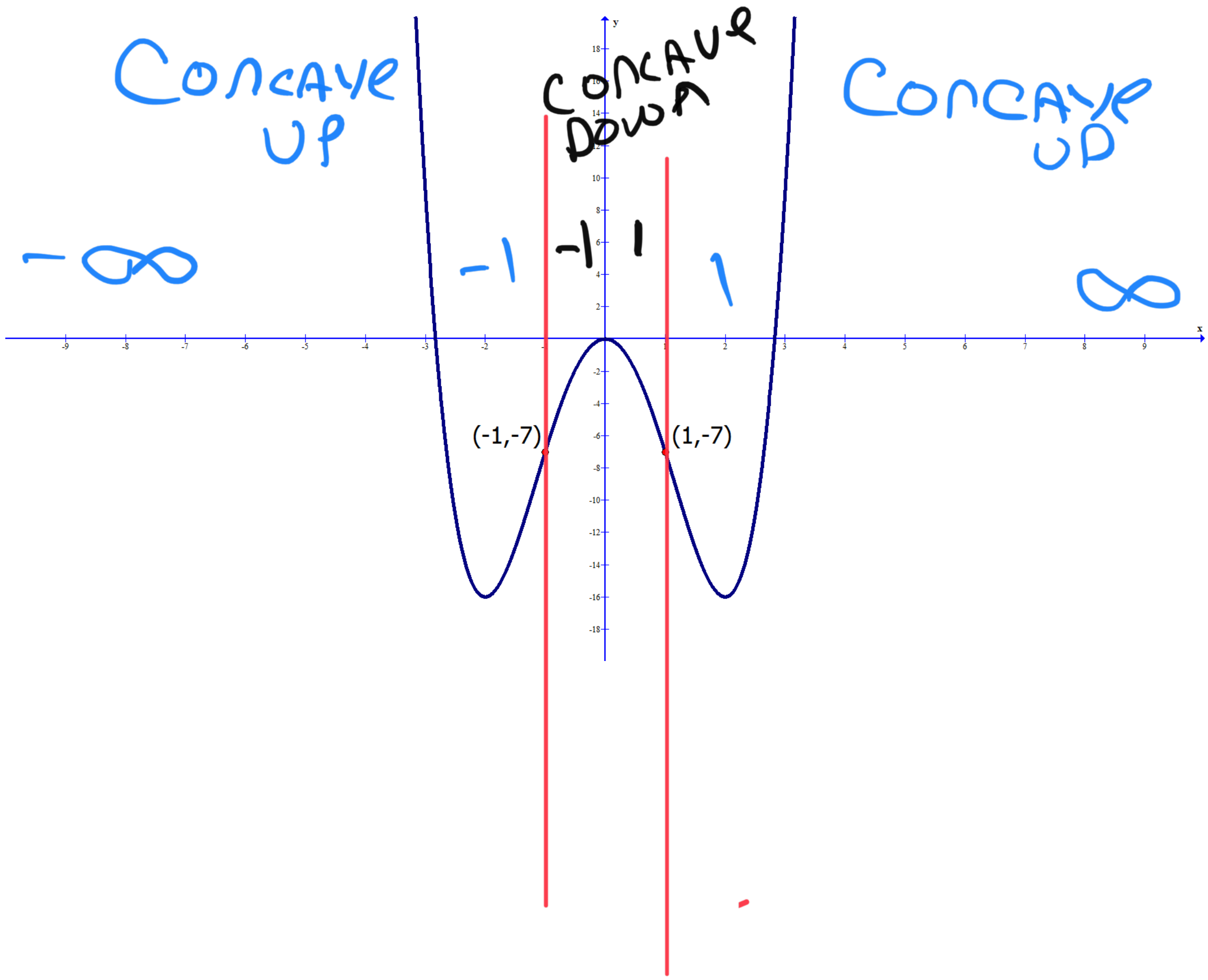


5a) Find the open interval(s) where the graph of the function is concave up $(-4,2)$

5b) Find the open interval(s) where the graph of the function is concave down. $(-\infty,-4) \cup (2,\infty)$

5c) Find all inflection points $(-4,384)$ *and* $(2,72)$

7)

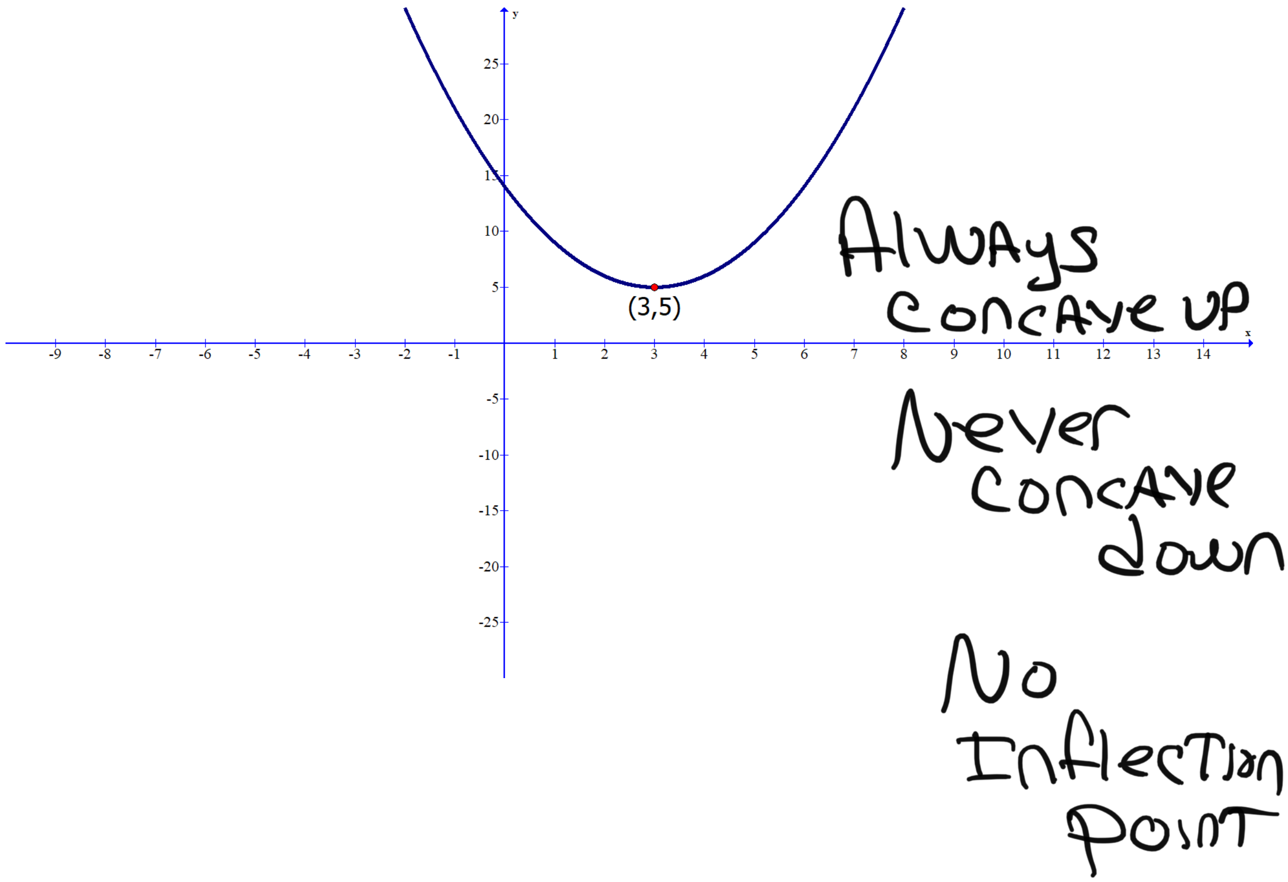


7a) Find the open interval(s) where the graph of the function is concave up $(-\infty,-1)\cup(1,\infty)$

7b) Find the open interval(s) where the graph of the function is concave down. $(-1,1)$

7c) Find all inflection points $(-1,-7)\ and\ (1,-7)$

9)

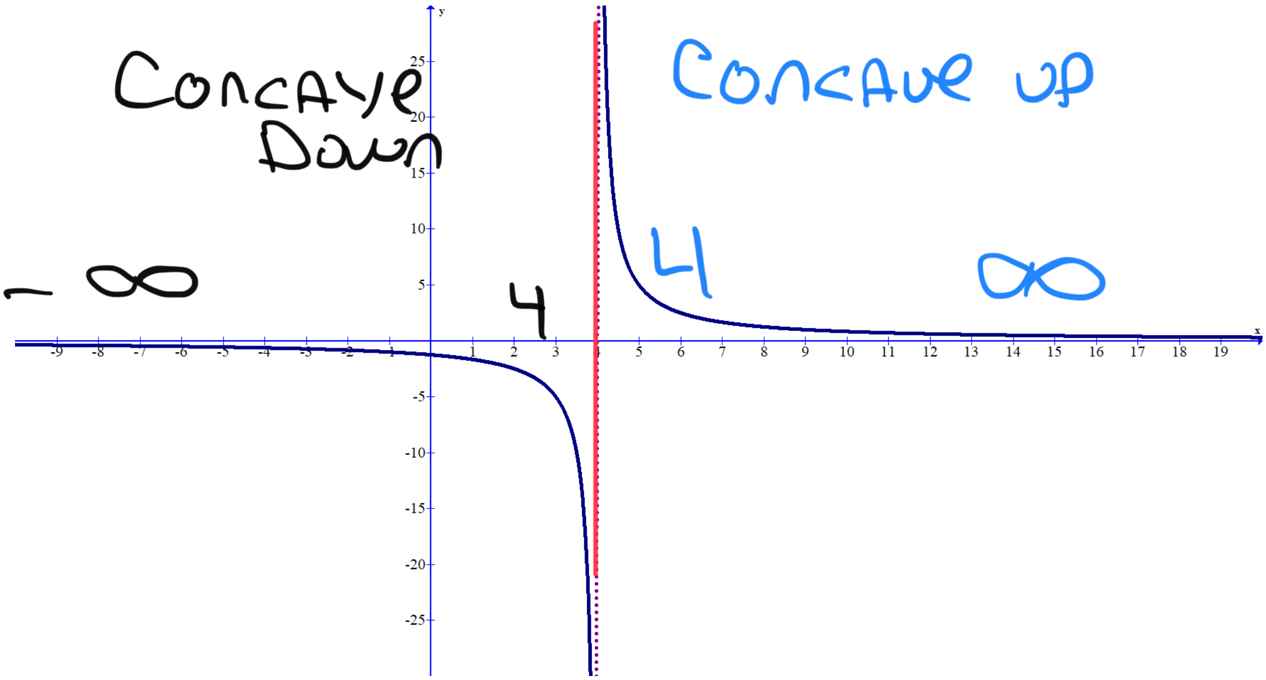


9a) Find the open interval(s) where the graph of the function is concave up $(-\infty, \infty)$

9b) Find the open interval(s) where the graph of the function is concave down. none

9c) Find all inflection points none

11)

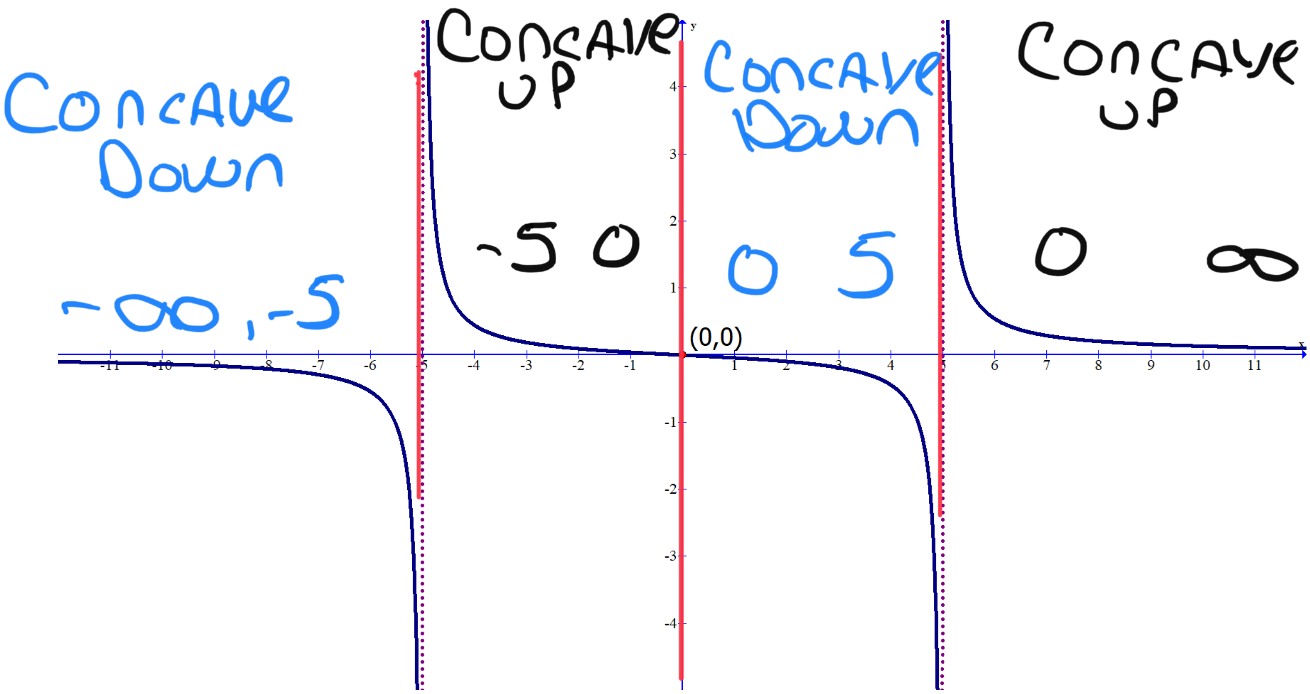


11a) Find the open interval(s) where the graph of the function is concave up $(4, \infty)$

11b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 4)$

11c) Find all inflection points $none, as\ x = 4\ is\ not\ in\ the\ domain\ of\ the\ function\ graphed$

13)



13a) Find the open interval(s) where the graph of the function is concave up $(-5,0) \cup (5, \infty)$

13b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, -5) \cup (0,5)$

13c) Find all inflection points $(0,0)$

#15-24:

a) Find the open interval(s) where the graph of the function is concave up

b) Find the open interval(s) where the graph of the function is concave down.

c) Find all inflection points

15) $f(x) = x^3 - 3x^2 + 5$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

$$6x - 6 = 0$$
$$6x = 6$$
$$x = 1$$

$-\infty$ 1 ∞

$x = 0$

$f''(0) = 6(0) - 6$

$= -6$

concave Down since negative

$x = 2$

$f''(2) = 6(2) - 6$

$= 6$

Concave up since positive

15a) Find the open interval(s) where the graph of the function is concave up $(1, \infty)$

15b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 1)$

15c) Find all inflection points $(1,3)$

y-coord Inflection point

$y = f(1) = 1^3 - 3(1)^2 + 5$

$= 3$

Point $(1,3)$

17) $f(x) = -x^3 - 3x^2 + 5$

$f'(x) = -3x^2 - 6x$

$f''(x) = -6x - 6$

$-6x - 6 = 0$

$-6x = 6$

$x = 6/-6 = -1$

$(-\infty, -1)$, $(-1, \infty)$

$-\infty$ $\quad -1 \quad$ ∞

$x = -2$

$f''(-1) = -6(-2) - 6$

$= 12 - 6$

$= 6$

Positive → concave up

$x = 0$

$f''(0) = -6(0) - 6$

$= -6$

negative → concave Down

17a) Find the open interval(s) where the graph of the function is concave up $(-\infty, -1)$

17b) Find the open interval(s) where the graph of the function is concave down. $(-1, \infty)$

17c) Find all inflection points $(-1,3)$

y-coord Inflection point

$y = f(-1) = -1(-1)^3 - 3(-1)^2 + 5$

$= 3$

Point $(-1,3)$

19) $f(x) = x^4 - 6x^2 + 4$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

$12x^2 - 12 = 0$

$12(x^2 - 1) = 0$

$12(x+1)(x-1) = 0$

$12 = 0$ No Sol. $\quad x + 1 = 0 \quad x = -1 \quad x - 1 = 0 \quad x = 1$

$-\infty \qquad -1 \qquad 1 \qquad \infty$

$x = -2$

$f''(-2) = 12(-2)^2 - 12 = 36$

Positive concave up

$x = 0$

$f''(0) = 12(0)^2 - 12 = -12$

Negative concave Down

$x = 2$

$f''(2) = 12(2)^2 - 12 = 36$

Positive concave up

19a) Find the open interval(s) where the graph of the function is concave up $(-\infty, -1) \cup (1, \infty)$

19b) Find the open interval(s) where the graph of the function is concave down. $(-1,1)$

19c) Find all inflection points $(-1,-1)$ *and* $(1,-1)$

Inflection points

y coord

$x = -1 \quad f(-1) = (-1)^4 - 6(-1)^2 + 4 = -1$

point $(-1,-1)$

$x = 1 \quad f(1) = (1)^4 - 6(1)^2 + 4 = -1$

point $(1,-1)$

21) f(x) = $2xe^x$

f'

First $2x$

Derivative 2

Second e^x

Derivative: $\frac{d}{dx}x * e^x = 1 * e^x = e^x$

$f'(x) = 2xe^x + 2e^x$

$f'(x) = 2e^x(x + 1)$

Easier to find $f"$ using the unfactored derivative $f'(x) = 2xe^x + 2e^x$

$f"(x) = (derivative\ 2xe^x) + (derivative\ of\ 2e^x)$

$f"(x)$ *same calculation as first derivative* $+ \; 2 * \frac{d}{dx}(x) * e^x$

$f"(x) = 2xe^x + 2e^x + 2e^x$

$f"(x) = 2xe^x + 4e^x$

$f"(x) = 2e^x(x + 2)$

$2e^x(x+2) = 0$

$2e^x = 0$

No sol

$x + 2 = 0$

$x = -2$

$-\infty \qquad -2 \qquad \infty$

Neg. concave down

$x = -3$

$f''(-3) = 2e^{-3}(-3+2)$

≈ -0.99

$x = 0$

$f''(0) = 2e^0(0+2)$

$= 2(1)(2)$

$= 4$

positive concave up

21a) Find the open interval(s) where the graph of the function is concave up $(-2, \infty)$

21b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, -2)$

21c) Find all inflection points $\left(-2, \frac{-4}{e^2}\right)$

y-coord Inflection point

$y = f(-2) = 2(-2)e^{-2}$

$= -4e^{-2} = \frac{-4}{e^2}$

Point $(-2, {-4}/{e^2})$

23) f(x) = $\frac{2}{x-5}$

Hint $f''(x) = \frac{4}{(x-5)^3}$

$$\frac{4}{(x-5)^3} = 0$$

$4 = 0$ No Sol

$(x-5)^3 = 0$

$x - 5 = 0$

$x = 5$

$-\infty$ 5 ∞

$x = 0$

$f''(0) = \frac{4}{(0-5)^3} = -.032$

concave down

$x = 6$

$f''(6) = \frac{4}{(6-5)^3} = 4$

concave up

23a) Find the open interval(s) where the graph of the function is concave up $(5, \infty)$

23b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 5)$

23c) Find all inflection points $none, as\ x = 5\ is\ not\ in\ the\ domain\ of\ the\ function$